If given Poisson distribution:

$$g(x;\lambda) = \frac{\lambda^x}{x!} exp\{-\lambda\}$$

The total likelihood

$$\mathcal{L}(D:\theta) = \prod_i^N \sum_k^K p_k g_k(D_i;\lambda_k)$$

$$\ell(D:\theta) = \log \prod_i^N \sum_k^K p_k g_k(D_i;\lambda_k) = \sum_i^N log \sum_k^K p_k g_k(D_i;\lambda_k)$$

Assume $Q(z^{(i)} = j) = P(z^{(i)} = j | x^{(i)}; \boldsymbol{p}, \boldsymbol{\lambda})$, which means the prob of $x^{(i)}$ belongs to j^{th} Poisson distribution with given $\boldsymbol{p}, \boldsymbol{\lambda}$**.**

We can add $Q(z^{(i)} = j)$ in to the likelihood by:

$$\ell(D:\theta) = \sum_i^N log \sum_k^K Q(z^{(i)} = k) \frac{p_k g_k(D_l;\lambda_k)}{Q(z^{(i)} = k)} \geq i$$

$$\ell(D:\theta) \geq \sum_i^N \sum_k^K Q(z^{(i)} = k) log \ \frac{p_k \frac{\lambda^x}{x!} exp\{-\lambda\}}{Q(z^{(i)} = k)}$$

according to the Jensen's theory.

Then for E step, still, we calculate the prob:

$$Q(z^{(i)} = j) = \frac{p_k g(x^{(i)}, \lambda_k)}{\sum_{l=1}^k p_k g(x^{(i)}, \lambda_l)}$$

In M step, we should maximize the likelihood by choose the $\boldsymbol{\phi}, \boldsymbol{\lambda}$, means that:

$$arg \max_{\boldsymbol{p},\boldsymbol{\lambda}} \ell(D, \boldsymbol{p}, \boldsymbol{\lambda})$$

Then we can have the minimum by chose the $\boldsymbol{p}, \boldsymbol{\lambda}$ where it's derivative equals 0

First, assuming

$$\omega_k^{(i)} = Q(z^{(i)} = k)$$

$$\sum_i^N \sum_k^K \omega_k^{(i)} log \ \frac{p_k \frac{\lambda_k^{x^{(i)}}}{x^{(i)}!} exp\{-\lambda_k\}}{\omega_k^{(i)}} \tag{1}$$

$$\nabla_{\lambda_k} \sum_i^N \sum_k^K \omega_k^{(i)} log \ \frac{p_k \frac{\lambda_k^{x^{(i)}}}{x^{(i)}!} exp\{-\lambda_k\}}{\omega_k^{(i)}} = 0$$

$$\nabla_{p_k} \sum_i^N \sum_k^K \omega_k^{(i)} log \; \frac{p_k \frac{{\lambda_k}^{x^{(i)}}}{x^{(i)}!} exp\{-\lambda_k\}}{\omega_k^{(i)}} = \; 0 \tag{2}$$

By solving equation (1), we can have

$$\nabla_{\lambda_k} \sum_i^N \sum_k^K \omega_k^{(i)} (x^{(i)} \log \lambda_k - \lambda_k) = \; 0$$

$$\sum_i^N \omega_k^{(i)} (\frac{x^{(i)}}{\lambda_k} - 1) = \; 0$$

$$\lambda_k = \; \frac{\sum_i^N \omega_k^{(i)} x^{(i)}}{\sum_i^N \omega_k^{(i)}}$$

By solving equation (2), we need to add normalization term

$$\sum_k^K p_k = 1$$

Then we should simplify the origin function with only terms has p_k:

$$\sum_i^N \sum_k^K \omega_k^{(i)} \log p_k$$

then use Lagrangian function

$$\mathcal{L}(p_k) = \; \sum_i^N \sum_k^K \omega_k^{(i)} \log p_k + \; \beta(\sum_k^K p_k - 1)$$

$$\frac{\partial \mathcal{L}(p_k)}{\partial p_k} = \; \sum_i^N \frac{\omega_k^{(i)}}{p_k} + \; \beta$$

And

$$p_k = \; \frac{\sum_i^N \omega_k^{(i)}}{\beta}$$

$$\sum_k^K p_k = 1 \; = \; \sum_k^K \frac{\sum_i^N \omega_k^{(i)}}{\beta}$$

$$\beta = N$$

So

$$p_k = \; \frac{\sum_i^N \omega_k^{(i)}}{N}$$